UNIVERSITY OF NOTRE DAME
DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING

Professor H.M. Atassi
113 Hessert Center
Tel: 631-5736
Email:atassi@nd.edu

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Advanced Aerodynamics

Homework 3

1. Consider the following fields:

$$u = x^2 + y^2 \quad v = tan^{-1}\frac{y}{x}, \tag{1}$$

$$u = \frac{cx}{x^2+y^2} \quad v = \frac{cy}{x^2+y^2}, \tag{2}$$

$$u = \frac{cy}{x^2+y^2} \quad v = -\frac{cx}{x^2+y^2}, \tag{3}$$

$$u = cx \quad v = -cy, \tag{4}$$

where c is a constant. Determine which of these fields can represent the velocity field of an incompressible flow and find the corresponding stream functions.

2. Suppose that the origin of coordinates, O, is a stagnation point in a two-dimensional flow. The the velocity vanishes there and hence the stream function, ψ satisfies,

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0.$$

Without loss of generality we may suppose $\psi = 0$ at the origin. Show that a Taylor expansion of ψ about the origin should be of the form

$$\psi = ax^2 + 2bxy + y^2 + \cdots$$

Deduce that the equation of the streamline in the vicinity of the origin is given by

$$ax^2 + 2bxy + y^2 = 0,$$

which represents two straight lines. Examine what happens if the lines are real or imaginary. Show that if the flow is irrotational, the two branches of the streamline cut at right angle.

3. Consider the velocity field where the x and y components of the velocity are given by

$$u = \frac{cx}{x^2+y^2} \tag{5}$$
$$v = \frac{cy}{x^2+y^2}, \tag{6}$$

where c is a constant. Is this field irrotational? Obtain the equation of the streamlines. Can you define a velocity potential? If yes, obtain the expressions for the velocity potential and the complex potential. What kind of flow does this field represent?

4. Consider the velocity field where the x and y components of the velocity are given by

$$u = \frac{cy}{x^2+y^2} \tag{7}$$
$$v = -\frac{cx}{x^2+y^2}, \tag{8}$$

where c is a constant. Is this field irrotational? Obtain the equation of the streamlines. Can you define a velocity potential? If yes, obtain the expressions for the velocity potential and the complex potential. What kind of flow does this field represent?

5. Consider the flow resulting from the superposition of a uniform stream V_∞ and a source-sink pair placed at a distance b to the left and right of the origin, respectively as shown in figure 1. The strength of the source and sink are Q and $-Q$, respectively. The stream function for the combined flow is given by

$$\psi = V_\infty r sin\theta + \frac{Q}{2\pi}(\theta_1 - \theta_2). \tag{9}$$

   (a) Express the stream function in terms of the $x - y$ coordinates.

   (b) Show that the stagnation points are located at

$$OA = OB = \sqrt{b^2 + \frac{Qb}{\pi V_\infty}}. \tag{10}$$

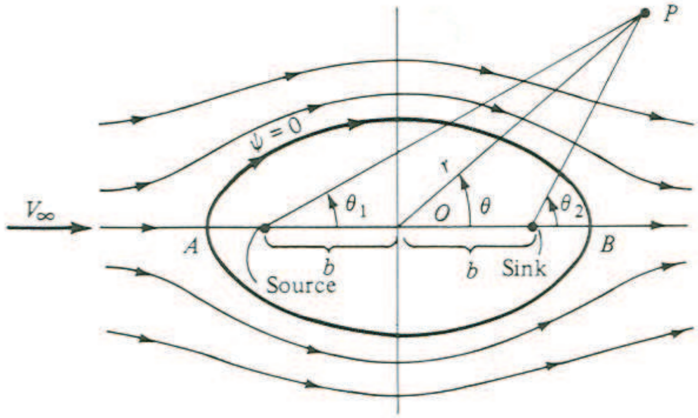

Figure 1: Flow over a Rankine oval: superposition of a uniform flow and a source-sink pair.

(c) Plot the stagnation streamline and show that (9) represents the flow about an oval. This is known as the Rankine oval flow. Plot a few streamlines around the oval for

$$b = 1, \quad \frac{Qb}{\pi V_\infty} = 0.1, 1, 10.$$

$$b = 0.1 \quad \frac{Qb}{\pi V_\infty} = 0.1, 1, 10.$$

6. Consider a flow with upstream velocity V_∞ about a circular cylinder of radius a in a uniform flow at angle of attack α with the x-axis. The flow around the cylinder has a circulation Γ. Develop a numerical scheme to plot the streamlines around the airfoil. Plot a few streamlines, including the stagnation streamlines. Consider the cases: (a) $|\Gamma/(4\pi a V_\infty)| < 1$, (b) $|\Gamma/(4\pi a V_\infty)| = 1$, and (c) $|\Gamma/(4\pi a V_\infty)| > 1$. Hint: Use the analytical expression for the velocity field given in class and a second order scheme for plotting the streamlines.